Temperature relaxation in two-temperature states of dense electron-ion systems

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It is shown that the Landau-Spitzer theory for temperature relaxation between electrons and ions, which was originally derived for ideal plasmas, is in fact more general. A relaxation formula is derived, for arbitrary ion-ion coupling that follows from elementary considerations combined with the fluctuation-dissipation theorem and the *f*-sum rule. The conditions for the validity of this theory are weak electron-ion coupling and that the spectrum of fluctuations of the ions lies at energies far below the resonances of the electrons spectrum. It is found that the rate of energy relaxation is not sensitive to the details of the ion-excitation spectrum. For classical electrons the formula reduces to the Landau-Spitzer form with minor modifications.

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I. INTRODUCTION

Matter in nature and in laboratory experiments is not always in thermodynamic equilibrium. The state behind a shock and the state generated by ultrashort pulse lasers are two well-known examples of nonequilibrium states. In the first case, the shock energy is imparted mainly to the ions. The energy transfer from the ions to the electrons, which eventually brings the system to a thermodynamic equilibrium, takes place downstream of the shock over a comparatively long time [1,2]. In the second example, laser energy couples mainly to the electrons. Also, in this case, equilibration between electron and ion temperatures takes place over a time much longer than the thermalization within each subsystem.

The theory of relaxation towards thermodynamic equilibrium in two-temperature plasmas was originally developed by Landau [3] and Spitzer [4]. The result was an explicit simple formula

$$\left(\frac{d}{dt}T_e\right)_{\text{Landau}} = n_0\,\frac{8}{3}\sqrt{2\pi}e^4\,\frac{1}{mM} \times\left\{\frac{(T_e-T_i)}{\left(\frac{T_i}{M}+\frac{T_e}{m}\right)^{3/2}}\right\}\int_{k_{\min}}^{k_{\max}}\frac{1}{k}\,dk. \qquad (1)$$

[T_e, (T_i), m, (M) are the electron (ion) temperature and mass, n_0 is the density]. Limiting the integration over the wave-vector k, between $k_{\min}$,$k_{\max}$ compensates for phenomena that diminish the effects of electron-ion collisions and are not included directly in the theory. Quantum interference and finite ion-size effects introduce an effective cutoff at large k. Coulomb screening introduces a small-k cutoff. This formula was obtained by summing the energy loss of electrons, via Coulomb collisions, using the Fokker-Planck (or Landau) kinetic equation. This approach applies only to ideal (weakly coupled) plasmas for which thermal energy of the particle exceeds by far the potential energy [5]. In principle, it does not apply to shocks in metals and liquids and to states generated by ultrashort pulse lasers, in which density is high and potential energy dominates over thermal kinetic energy.

In a recent work [6], Dharma-wardana and Perrot presented formulas for the rate of energy relaxation. In the analysis, they made a distinction between the above-mentioned class of ''single-particle'' or ''Spitzer-type'' formulas for energy relaxation and a more general class of formulas. The more general formulas, appropriate also for nonideal plasmas with strong electron-electron, ion-ion, and electron-ion coupling, account for relaxation via interaction of normal modes of the hot and cold subsystems.

In the present paper, we will show that for weak electron-ion coupling, but possibly strong ion-ion coupling, and when the ion-excitation spectrum lies well below the electron-excitation spectrum, the formula for the rate of energy relaxation may be obtained using elementary considerations combined with the fluctuation-dissipation theorem [7]. The summation over all mode frequencies required by the formula is obtained analytically using the *f*-sum rule [8]. Surprisingly, the rate of temperature relaxation does not depend on the details of the ionic spectrum of excitations $S^{\text{II}}(k,\omega)$ and depends only on the low-frequency ($\omega\sim 0$) properties of the electronic spectrum of fluctuations, $S^{\text{ee}}(k,\omega)$. When the classical limit is taken, the final formula reduces to the Landau form, with minor modifications.

This result shows that the Landau theory [3,4], and its extensions [9] for temperature relaxation in weakly coupled plasmas, can in fact be more general. It applies, with minor modifications, also to systems with strong ion-ion coupling. The condition for the validity of this theory is that electron-ion coupling is weak and the spectrum of fluctuations of the ions lies far below the resonances in the electrons spectrum.

II. DERIVATION OF THE FORMULA FOR THE RATE OF ENERGY RELAXATION BY USING THE FLUCTUATION DISSIPATION THEOREM

The transfer of energy between the electron and ion subsystems is due to field fluctuations in one subsystem acting on the current fluctuations in the other subsystem. Even for systems with strong ion-ion coupling, the analysis of the

energy-transfer process becomes tractable provided that the electron-ion coupling is weak.

Consider a disturbance (of wave-vector $\vec{k}$ and frequency ω), $\delta n^{I}_{\vec{k},\omega}$, in the ion density. This disturbance may be characterized also by the potential viewed by the electrons:

$$\delta\phi^{I}_{\vec{k},\omega} = V^{\mathrm{ie}}_{\vec{k}}\,\delta n^{I}_{\vec{k},\omega}. \tag{2}$$

For the simple case where the ions may be treated as points of charge, Eq. (2) is just the Coulomb law with $V^{\mathrm{ie}}_{\vec{k}} = e^2/k^2$. We have used the factor $V^{\mathrm{ie}}_{\vec{k}}$ to allow for a more general treatment in which the interaction between electrons and ions may be described by a pseudopotential [10] that accounts also for the finite size of the ion core.

Since the pseudopotential $V^{\mathrm{ie}}_{\vec{k}}$ is weak by construction, we may assume that the total system is made up of two subsystems with the response of each subsystem given by a generalized mean-field-type expression of the form

$$\chi^{\mathrm{ee}}(\vec{k},\omega) = \chi^{\mathrm{ee}}(\vec{k},\omega)^0/(1-e^2V_k)(1-G^{\mathrm{ee}}_k) \times \chi^{\mathrm{ee}}(\vec{k},\omega)^0\chi^{\mathrm{ii}}(\vec{k},\omega),$$

$$\chi^{\mathrm{ii}}(\vec{k},\omega) = \chi^{\mathrm{ii}}(\vec{k},\omega)^0/(1-(\mathrm{Ze})^2V_k(1-G^{\mathrm{ii}}_k)\chi^{\mathrm{ii}}(\vec{k},\omega)^0. \tag{3}$$

Here, the noninteracting response function (e.g., for electrons) is denoted by $\chi^{\mathrm{ee}}(\vec{k},\omega)^0$, while G^{ee}_k is the corresponding local-field correction.

Within the framework of linear-response theory, the disturbance $\delta\phi^{I}_{\vec{k},\omega}$ induces an electron response that may be characterized by the disturbed density $\delta n^{\mathrm{ind},e}_{\vec{k},\omega}$ which is related to the ion disturbance by

$$\delta n^{\mathrm{ind},e}_{\vec{k},\omega} = \delta\phi^{I}_{\vec{k},\omega}\chi^{\mathrm{ee}}(\vec{k},\omega) \tag{4}$$

and a corresponding current

$$\frac{\vec{k}}{k}\cdot\delta\vec{J}^{\mathrm{ind},e}_{\vec{k},\omega} = e\,\delta n^{\mathrm{ind},e}_{\vec{k},\omega}\left(\frac{\omega}{k}\right) = e\,\delta\phi^{I}_{\vec{k},\omega}\left(\frac{\omega}{k}\right)\chi^{\mathrm{ee}}(\vec{k},\omega). \tag{5}$$

The energy transfer between the electrons and the ions due to an ensemble of such ion-density fluctuations is

$$\begin{aligned}\frac{d}{dt}K^I &= \int \frac{d^3\vec{k}}{(2\pi)^3}\,d\omega e\ \mathrm{Im}\langle \delta\vec{J}^{\mathrm{ind},e}_{\vec{k},\omega}\cdot\delta\vec{E}^{I}_{-\vec{k},-\omega}\rangle \\ &= \int \frac{d^3\vec{k}}{(2\pi)^3}\,d\omega e^2\omega(V^{\mathrm{ie}}_{\vec{k}})^2\,\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega) \times\langle \delta n^{I}_{\vec{k},\omega}\delta n^{I}_{-\vec{k},-\omega}\rangle] \\ &= -\int \frac{d^3\vec{k}}{(2\pi)^3}\,d\omega\,\frac{\hbar}{2\pi}\coth\left(\frac{\hbar\omega}{2T_i}\right)e^2\omega(V^{\mathrm{ie}}_{\vec{k}})^2 \times\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega)]\mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)],\end{aligned} \tag{6}$$

where $\langle\cdots\rangle$ denotes the equilibrium-ensemble average and we have used the fluctuation-dissipation theorem [7]:

$$S^{\mathrm{II}}(\vec{k},\omega) = -\frac{\hbar}{2\pi}\coth\left(\frac{\hbar\omega}{2T_e}\right)\mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)], \tag{7}$$

which connects the spectrum of fluctuations $S^{\mathrm{II}}(\vec{k},\omega) = \langle \delta n^{I}_{\vec{k},\omega}\delta n^{I}_{-\vec{k},-\omega}\rangle$ and the density response function $\chi^{\mathrm{II}}(\vec{k},\omega)$.

Note that in our derivation of Eq. (5) $\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega)]$ comes from the electron response while $\mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)]$ comes from the averaging process $\langle \delta n^{I}_{\vec{k},\omega}\delta n^{I}_{-\vec{k},-\omega}\rangle$ with the aid of the fluctuation-dissipation theorem, i.e., the derivation of Eq. (6) involves only the averaging over ion fluctuations

Similarly, a fluctuation in the electron density, $\delta n^{e}_{\vec{k},\omega}$, with the corresponding potential

$$\delta\phi^{e}_{\vec{k},\omega} = V^{\mathrm{ie}}_{\vec{k}}\,\delta n^{e}_{\vec{k},\omega} \tag{8}$$

will induce a perturbation in the ion density and current

$$\delta n^{\mathrm{ind},I}_{\vec{k},\omega} = \delta\phi^{e}_{\vec{k},\omega}\chi^{\mathrm{II}}(\vec{k},\omega), \tag{9}$$

$$\frac{\vec{k}}{k}\cdot\delta\vec{J}^{\mathrm{ind},I}_{\vec{k},\omega} = -e\,\delta n^{\mathrm{ind},I}_{\vec{k},\omega}\left(\frac{\omega}{k}\right) = -e\,\delta\phi^{e}_{\vec{k},\omega}\left(\frac{\omega}{k}\right)\chi^{\mathrm{II}}(\vec{k},\omega). \tag{10}$$

The energy transfer between the electrons and ions due to an ensemble of such electron-density fluctuations is

$$\begin{aligned}\frac{d}{dt}K^e &= \int \frac{d^3\vec{k}}{(2\pi)^3}\,d\omega e\ \mathrm{Im}\langle \delta\vec{J}^{\mathrm{ind},I}_{\vec{k},\omega}\cdot\delta\vec{E}^{e}_{-\vec{k},-\omega}\rangle \\ &= \int \frac{d^3\vec{k}}{(2\pi)^3}\,d\omega\,\frac{\hbar}{2\pi}\coth\left(\frac{\hbar\omega}{2T_e}\right)e^2\omega(V^{\mathrm{ie}}_{\vec{k}})^2 \times\mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)]\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega)].\end{aligned} \tag{11}$$

The total energy transfer between the electrons and ions is the sum of transfer due to the interaction between spontaneous fluctuations in ion density and the corresponding induced perturbation in the electron density, and the transfer due to the interaction between the spontaneous fluctuations in electron density and the corresponding induced perturbation in the ion density, i.e.,

$$\begin{aligned}\frac{d}{dt}K &= \frac{d}{dt}K^e + \frac{d}{dt}K^I \\ &= \int \frac{d^3\vec{k}}{(2\pi)^3}\,\omega d\omega\,\frac{\hbar}{2\pi}\left[\coth\left(\frac{\hbar\omega}{2T_e}\right)-\coth\left(\frac{\hbar\omega}{2T_i}\right)\right] \times(eV^{\mathrm{ie}}_{\vec{k}})^2\,\mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)]\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega)].\end{aligned} \tag{12}$$

This equation is equivalent to Eq. (36) in Ref. [6] where it was derived using a variant of the Fermi golden rule.

Note that the only contributions to energy transfer between the electron and ion subsystems come from interaction between fields and the corresponding *induced* currents. These are the only in-phase terms. Other terms are averaged out due to the statistical isotropy of the random fields and currents.

Note also that the derivation of Eq. (12) is based on separation between fluctuations in one subsystem (electrons or ions) which are the result of all the interactions and correlations within this subsystem and the corresponding induced fluctuations in the other subsystem. Back effect of the induced fluctuations are not considered. This is equivalent to the lowest significant order in an expansion with the electron-ion coupling as a small parameter, while ion-ion coupling may be of arbitrary strength (as long as it is consistent with the magnitude of the e-I coupling parameter).

III. EVALUATION OF THE RELAXATION RATE BY USING THE *f*-SUM RULE

The spectrum of fluctuations, $\sim \mathrm{Im}[\chi^{\mathrm{II}}(\vec{k},\omega)]$ drops to zero very rapidly as the phase velocity ω/k of the excitation exceeds the ions thermal velocity, $\sqrt{T_i/M}$, or as the frequency ω exceeds the ion plasma frequency $\omega_{\mathrm{pi}} = \sqrt{4\pi n_0 e^2/M}$. In most cases, for the electrons, this is a very low-frequency range, i.e., the range that contributes to the ω integration in Eq. (12) is characterized by $\omega \ll k\sqrt{T_e/m}$ and also $\hbar\omega/2T_i, \hbar\omega/2T_e \ll 1$.

Using this feature [12] we may replace $(\hbar\omega/2)\coth(\hbar\omega/2T)$ by T and $\mathrm{Im}[\chi^{\mathrm{ee}}]$, by its low-frequency limit

$$\mathrm{Im}[\chi^{\mathrm{ee}}(\vec{k},\omega)] \sim \omega\, \mathrm{Im}\left[\frac{\partial}{\partial\omega}\chi^{\mathrm{ee}}(\vec{k},\omega)\right]_{\omega=0}, \tag{13}$$

(where we have used the antisymmetry of χ^{ee} with respect to ω). Representing the pseudopotential as $eV^{\mathrm{ie}}_{\vec{k}} = (4\pi e^2/k^2)M_{\vec{k}}$, Eq. (12) is reduced to

$$\frac{d}{dt}K = \int \frac{1}{\pi}[T_e - T_i]\frac{4\pi e^2}{k^2}|M_{\vec{k}}|^2\, \mathrm{Im}\left[\frac{\partial}{\partial\omega}\chi^{\mathrm{ee}}(\vec{k},\omega)\right]_{\omega=0} \times\left\{\int_{-\infty}^{\infty}\omega\, \mathrm{Im}\left[\frac{4\pi e^2}{k^2}\chi^{\mathrm{II}}(\vec{k},\omega)\right]d\omega\right\}\frac{d^3\vec{k}}{(2\pi)^3}. \tag{14}$$

By the *f*-sum rule for the ions (just another manifestation of conservation of particles [8]), no matter what is the shape of the ion response function (which may also include the electron screening effect) the ω integration in the curled brackets in Eq. (14) depends only on the ion density and equals $-\pi\omega_{\mathrm{pi}}^2$.

Using this result, one gets the following simplified formula for the energy relaxation rate:

$$\frac{d}{dt}K = \omega_{\mathrm{pi}}^2[T_e - T_i] \times\int \frac{4\pi e^2}{k^2}|M_{\vec{k}}|^2\, \mathrm{Im}\left[\frac{\partial}{\partial\omega}\chi^{\mathrm{ee}}(\vec{k},\omega)\right]_{\omega=0}\frac{d^3\vec{k}}{(2\pi)^3}. \tag{15}$$

According to Eq. (15), the rate of energy relaxation between electrons and ions is not sensitive to the details of the ionic spectrum of fluctuations. The physical explanation of this feature is very simple. In the relevant range of ω, the spectrum of fluctuations of the electron density is independent of ω, thus, the overlap of the electron-ion spectra is proportional to the total energy in the ionic spectrum of fluctuations, which by the *f*-sum rule, depends only on the ion density. This means that, in spite of the independence on ion spectrum, formula (15) already includes relaxation by interaction between electrons and all ion-collective and single-particle modes.

It should be emphasized that this is a direct consequence of the large separation (in $\omega,\vec{k}$ space) between the spectra of the electrons and ions. In systems for which coupled modes dominate [11], the representation used in the present paper of distinct electron fluctuation spectrum and ion fluctuation spectrum, each obeying the fluctuation-dissipation theorem and the *f*-sum rule separately, is not valid.

IV. REDUCTION TO THE LANDAU-SPITZER FORM

In order to make contact with the simplified result [Eq. (1)] of the Landau theory, let us consider a rather general and widely used mean-field approximation, for the electron-response function, with local-field correction [10],

$$\chi^{\mathrm{ee}} = \frac{\chi^{0e}}{1 - eV^{\mathrm{ee}}_{\vec{k}}(1 - G^{\mathrm{ee}}_k)\chi^{0e}}, \tag{16}$$

where $\chi^{0e}(k,\omega)$ is the density response function for noninteracting Fermi gas [8], G^{ee}_k is the (static) local-field correction and $V^{\mathrm{ee}}_{\vec{k}}$ is the potential acting between electrons. The ω derivative at the origin reads

$$\left[\frac{\partial}{\partial\omega}\chi^{\mathrm{ee}}\right]_{\omega=0} = \left[\frac{\frac{\partial}{\partial\omega}\chi^{0e}}{(1 - eV^{\mathrm{ee}}_{\vec{k}}(1 - G^{\mathrm{ee}}_k)\mathrm{Re}(\chi^{0e}))^2}\right]_{\omega=0}, \tag{17}$$

where we have used, again, the antisymmetry of $\mathrm{Im}(\chi^{0e})$ with respect to ω. With this result, the energy relaxation rate of Eq. (15) is given by

$$\frac{d}{dt}K=\omega_{\rm pi}^2[T_e-T_i]\int\frac{4\pi e^2}{k^2}|M_{\vec{k}}|^2 \times\frac{\mathrm{Im}\left[\frac{\partial}{\partial\omega}\chi^{0e}(\vec{k},\omega)\right]_{\omega=0}}{(1-eV_{\vec{k}}^{\rm ee}(1-G_k^{\rm ee})\mathrm{Re}(\chi^{0e}(k,0))^2}\frac{d^3\vec{k}}{(2\pi)^3}. \tag{18}$$

Formula (18) is equivalent to formula (1) and its quantum generalization presented in Ref. [9]. To demonstrate this, let us use the response function of nondegenerate electrons, i.e.,

$$\mathrm{Re}[\chi^{0e}(k,0)]=-\frac{n_0}{T_e}, \tag{19}$$

$$\mathrm{Im}[\chi^0(k,\omega)]=-\frac{n_0}{T_e}\sqrt{\frac{\pi}{2}}\frac{\omega}{k\sqrt{\frac{T}{m}}}e^{-(\omega^2/2(T/m)k^2+\hbar^2k^2/8mT)} \times\frac{\sinh\left(\frac{\hbar\omega}{T}\right)}{\frac{\hbar\omega}{T}}, \tag{20}$$

and

$$\mathrm{Im}\left[\frac{\partial}{\partial\omega}\chi^{0e}(\vec{k},\omega)\right]_{\omega=0}=-\frac{n_0}{T_e}\sqrt{\frac{\pi}{2}}\frac{e^{-\hbar^2k^2/8mT_e}}{k\sqrt{T_e/m}}. \tag{21}$$

Taking $eV_{\vec{k}}^{\rm ee}=4\pi e^2/k^2$ in Eq. (18), we get the following formula for the temperature relaxation rate:

$$\frac{d}{dt}T_e=\frac{2}{3}\frac{1}{n_0}\frac{d}{dt}K=\sqrt{2\pi}\frac{8}{3}n_0\frac{e^4}{Mm}\frac{[T_e-T_i]}{(T_e/m)^{3/2}} \times\int|M_{\vec{k}}|^2\frac{k^4e^{-\hbar^2k^2/8mT_e}}{[k^2+k_D^2(1-G_k^{\rm ee})]^2}\frac{1}{k}dk. \tag{22}$$

Note that in Eq. (22), the range of wave-vectors k that contribute to the integral are limited by the denominator and by the exponent to the range: $\sqrt{k_D^2(1-G_k^{\rm ee})}=k_{\rm min}<k<k_{\rm max}=\sqrt{8mT_e/\hbar^2}$. This equation was derived with the restriction $T_e/m\gg T_i/M$, so that Eq. (22), with $|M_{\vec{k}}|^2=1$, is essentially the Landau-Spitzer result [Eq. (1)].

We have considered singly ionized plasmas. For a plasma with *ion* density n_0 and a degree of ionization Z, the rate of energy relaxation is larger by a factor of Z^2 than the value predicted by Eqs. (1), (12), and (22).

V. NUMERICAL DEMONSTRATION

In order to demonstrate the validity of the various approximations applied, we calculate the energy relaxation per ion for aluminum plasma at normal density and with the ions at its melting point ($T_i=0.0813$ eV). The pseudopotentials are as in Ref. [6]. Here, the ionization changes from $Z=3$ to $Z=5.2$ when T_e varies from 2 to 40 eV. Figure 1 compares the rate of energy relaxation as predicted by the three approximations of the "mode overlap" formulas, Eq. (12), which are labeled as follows: "All-ω" refers to a direct integration as prescribed in the "mode-overlap" formula [Eq. (12)]; "f-sum" is the low-frequency approximation+f-sum rule result [Eq. (15)]; "Cls-A" refers to a calculation according to Eq. (22) using the same pseudopotentials and local-field corrections as in the "All-ω" and "f-sum" calculations. These result show that the low-frequency+f-sum rule approximation presents a fairly good estimate to the rate of energy relaxation as predicted by the "mode-overlap" formula (12). Also, the "Cls" line, which follows formula (22) which is very similar to the Landau-Spitzer formula, is fairly close to the "All-ω" line.

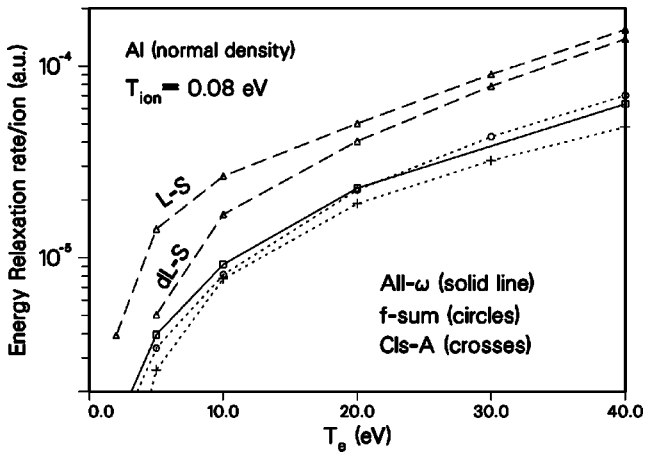

FIG. 1. Rate of energy relaxation per ion (atomic units) as a function of electron temperature (eV) in various approximations: "All-ω" - a direct integration as prescribed in the "mode overlap" formula [Eq. (12)]; "f-sum" the low-frequency approximation + f-sum rule result [Eq. (15)]; "Cls-A" a calculation according to Eq. (22) using the same pseudopotentials and local-field corrections as in the "All-ω" and "f-sum" calculations. "L-S" -Landau-Spitzer formula; "dL-S" -a generalization that treats also electron degeneracy as in Ref. [9] but with nonzero ion temperature. The cutoff wave vectors, $k_{\rm min}k_{\rm max}$ necessary for formulas "L-S" and "dL-S" were taken as in Ref. [13].

On top of these lines we have added two lines that correspond to a direct evaluation of the Landau-Spitzer formula (labeled "L-S") and a generalization that treats also electron degeneracy [9] (labeled "dL-S"). The cutoff wave vectors $k_{\rm min}$,$k_{\rm max}$ necessary for these formulas were taken as in Ref. [13]. From these results, we see that even a direct use of the Landau-Spitzer formula (as derived for weakly coupled plasmas, without modifications) gives an estimate of the relaxation rate that is within a factor of 2–3 the same as the prediction of the "mode-overlap" formula.

VI. SUMMARY

We have derived a simple formula [Eq. (15)] for the rate of energy transfer between electrons and ions in two-

temperature systems, possibly with strong ion-ion and electron-electron coupling. The condition for the validity of this formula is that electron-ion coupling is weak and the spectrum of fluctuations of the ions lies far below the resonances in the electrons spectrum. According to this formula, the rate of energy relaxation between electrons and ions is not sensitive to the details of the ionic spectrum of fluctuations and depends only on the near-zero frequency ($\omega \sim 0$) properties of the electronic spectrum of fluctuations. The physical explanation of this feature is that in the relevant range of ω, the spectrum of fluctuations of electron density is independent of ω, thus, the overlap integral of the electron-ion spectra is proportional to the total energy in the ionic spectrum of fluctuations, which by the *f*-sum rule, depends only on the ion density. This means that, in spite of the independence on ion spectrum, the formula already includes relaxation by interaction between electrons and all ion (collective- and single-particle) modes. The well-known Landau-Spitzer formula [3,4] [Eq. (1)], is recovered, with minor modifications, when the electronic-response function near zero frequency is replaced by its random-phase approximation with a local-field correction [10] [see Eq. (22)]. This shows that the Landau theory that was originally derived for ideal plasmas, is in fact more general and applies, with minor modifications, also to systems with strong ion-ion coupling. That is, one may use the Landau-Spitzer formula, in experimental situations where electrons are much hotter than ions, and such that the electron-ion coupling is weak.

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